Total Number of Questions : 30

Time : 2.00 Hours

Max. Marks : 100

1. Mention the principles of programmed instruction. **(2 Marks)**

2. Explain the concept of TPCK. **(2 Marks)**

3. 'Mathematics knowledge is a priori'. Comment. **(2 Marks)**

4. 'Spiral approach is most recommended in organizing school Mathematics curriculum'. Why ? **(2 Marks)**

5. What are the major developments in Mathematics in 20^{th} century ? **(2 Marks)**

6. Bringout the importance of Link Practice in the training of teaching skills. **(2 Marks)**

7. What are the process abilities in Mathematics that are expected to be developed among students by learning Mathematics ? **(2 Marks)**

8. Citing an example from Mathematics establish Bruner's statement 'The foundations of any subject may be taught to anybody at any age in some form'. **(2 Marks)**

9. Does there exist a linear transformation $T : \mathbb{R}^{2021} \to \mathbb{R}^{2021}$ such that the range and null space of T are identical ? Justify your claim. **(3 Marks)**

10. Suppose that $x_n \geq 0$ and $\lim ((-1)^n x_n)$ exists. Show that (x_n) converges. **(3 Marks)**

11. If f is continuous on [0, 1] and if $\int_0^1 f(x)x^n dx = 0$ for n = 0, 1, 2, . . ., then show that f(x) = 0 for every $x \in [0, 1]$. **(3 Marks)**

12. Let X be a topological space. Show that every subset of X is open if and only if each subset containing a single point is open. **(3 Marks)**

13. Show by an example that a non-zero prime ideal of a commutative ring with unity need not be maximal. **(3 Marks)**

14. Is $10 = (3 + i) \times (3 + i) = 2 \times 5$ an example of non-unique factorization in $\mathbb{Z}[i]$? Justify your claim. **(3 Marks)**

15. Suppose f(z) and g(z) are entire functions, g(z) is never zero and $|f(z)| \leq |g(z)|$ for all z. Show that there is a constant c such that f(z) = cg(z). **(3 Marks)**

16. Consider the real vector space V of polynomials of degree less than or equal to n. For $P \in V$ define $\|P\|_K = \max\{|P(0)|, |P^{(1)}(0)|, \ldots, |, P^{(K)}(0)|\}$, $P^{(i)}$ denotes the i^{th} derivative of P. Show that $\| \ \|_K$ denotes a norm iff $K \geq n$. **(3 Marks)**

17. Show that the initial-value problem $y' = \dfrac{4t^3 y}{1+t^4}$, $0 \leq t \leq 1$, y(0) = 1 has a unique solution, and find the solution. **(3 Marks)**

18. Solve the congruence $25x \equiv 15 \pmod{120}$. **(3 Marks)**

19. Explain the knowledge dimensions in Revised Bloom Taxonomy with suitable examples from Mathematics. **(3 Marks)**

20. Elucidate the need of diagnostic testing and remedial teaching in Mathematics. **(3 Marks)**

21. Differentiate between Holistic rubric and Analytic rubric. **(3 Marks)**

22. Let V be the space of all real-valued continuous functions defined on $\mathbb{R}$ and $T : V \to V$ be defined by $(Tf)(x) = \int_0^x f(t)\,dt; f \in V, x \in \mathbb{R}$. Show that T has no eigenvalues. **(5 Marks)**

23. Let μ be the Lebesgue measure, $f \geq 0$ and $\int_E f d\mu = 0$. Show that $f = 0$ almost everywhere on E. **(5 Marks)**

24. Show that if X is a compact metric space, then X is separable. **(5 Marks)**

25. Prove or disprove. The symmetric group S_n has exactly 3 normal subgroups for $n \geq 5$. **(5 Marks)**

26. Let f(z) be a non constant analytic function defined in a domain $D = \{z : |z| < r\}$ and continuous on 2D such that $|f(z)| > m$ on 2D. If $|f(0)| < m$, show that there exists atleast one zero in $|z| < r$. **(5 Marks)**

27. Suppose X and Y are F-spaces, $\wedge : X \to Y$ is linear and $G = \{(x, \wedge(x)) : x \in X\}$ is closed in $X \times Y$. Then show that $\wedge$ is continuous. **(5 Marks)**

28. Solve the PDE $\left(y^2 + z^2\right)\frac{\partial z}{\partial x} - xy\frac{\partial z}{\partial y} + xz = 0$. **(5 Marks)**

29. If a prime p does not divide a then prove that $a^{p-1} \equiv 1 \pmod{p}$. **(5 Marks)**

30. Explain briefly the cognitive development model. **(5 Marks)**